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Variational integrators for the dynamics of thermo-elastic solids with finite speed thermal waves ☆,☆☆

Pablo Mata [a,b], Adrian J. Lew [a,*]

[a] Department of Mechanical Engineering, Stanford University, Stanford, CA 94305-4040, USA
[b] Centro de Investigación en Ecosistemas de la Patagonia (CIEP), Conicyt Regional/CIEP R10C1003, Universidad Austral de Chile, Ignacio Serrrano 509, Coyhaique, Chile

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ABSTRACT

This paper formulates variational integrators for finite element discretizations of deformable bodies with heat conduction in the form of finite speed thermal waves. The cornerstone of the construction consists in taking advantage of the fact that the Green–Naghdi theory of type II for thermo-elastic solids has a Hamiltonian structure. Thus, standard techniques to construct variational integrators can be applied to finite element discretizations of the problem. The resulting discrete-in-time trajectories are then consistent with the laws of thermodynamics for these systems: for an isolated system, they exactly conserve the total entropy, and nearly exactly conserve the total energy over exponentially long periods of time. Moreover, linear and angular momenta are also exactly conserved whenever the exact system does. For definiteness, we construct an explicit second-order accurate algorithm for affine tetrahedral elements in two and three dimensions, and demonstrate its performance with numerical examples.

1. Introduction

The classical treatment to describe heat conduction in solids adopts Fourier's law to describe the heat flux (e.g., [1–3]). In the context of thermo-elastic materials, this assumption leads to the transmission of thermal disturbances with infinite speed [4,5]. Notwithstanding, this model has been found to be very useful in many engineering applications. On the other hand, since several decades ago (e.g. [6]) it has been recognized that heat transport with finite speed thermal waves may be useful in modeling some materials at low temperatures. This phenomenon is frequently mentioned in the literature as *second sound*, to differentiate it from the *first sound* in solids, which is related to the propagation of mechanical waves. Other contexts in which such theories could be useful are referred to in [7–10].

A number of theories have been proposed to model heat transfer with finite speed thermal waves, e.g., [4,11]. Among them, some incorporate a regularization of the heat flux to include a relaxation time controlling the velocity of propagation of heat perturbations [12,13], see [14] for a thorough discussion. A variational theory for heat conduction in rigid solids with finite wave speed based on a non-autonomous Hamiltonian was considered in [15–17]. This approach recovers the classical Fourier case in the limit in which a relaxation parameter approaches zero. An attempt to extend these ideas to general problems of non-conservative type is described in [18], and a survey of such models can be found in [19]. Alternative

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* Corresponding author.
E-mail address: lewa@stanford.edu (A.J. Lew).

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