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Explicit symplectic momentum-conserving time-stepping scheme for the dynamics of geometrically exact rods

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ABSTRACT

A new structure-preserving algorithm for simulating the nonlinear dynamics of geometrically exact rods is developed. The method is based on the simultaneous discretization in space and time of Hamilton's variational principle. The resulting variational integrator is explicit, second-order accurate and can be identified with a Lie-group symplectic partitioned Runge–Kutta method for finite element discretizations of rods involving large rotations and displacements. Numerical examples allow to verify that the algorithm presents an excellent long term energy behavior along with the exact conservation of the momenta associated to the symmetries of the system.

1. Introduction

The formulation of continuum based theories for rods has captured the interest of researchers during decades [60]. The motivation for developing such theories is rooted in attempting to model complex physical problems arising in the dynamics of slender solids by means of a dimensionally reduced approach. This point of view has boosted the construction of models depending on one spatial variable and the time [1,73].

One of the most successful models[1] for describing the dynamics of elastic rods undergoing finite deformation has been proposed by Simo in [65]. This geometrically exact formulation follows a director-type approach to describe the kinematics of the rods and corresponds to a three-dimensional generalization of Reissner's original model [60,61]. In a posterior work, Simo and Vu-Quoc [69,70] proposed a numerical scheme based on combining the finite element method (FEM) with a modified version of Newmark's scheme for time integration. Other authors have also made highlighting contributions. For example, an initially curved reference configuration for the rod has been considered in [31,32,37]. A total Lagrangian formulation has been proposed in [11] and revisited in [47]. Some important numerical issues such as the construction of non-locking elements or frame-indifferent formulations can be consulted in [8,35,62,63]. An alternative formulation based on some concepts of geometric (Clifford) algebra is presented in [58]. The list of works largely exceeds the mentioned ones, which only constitute some relevant examples; a more complete survey can be consulted in [53].

From the point of view of the applications, this formulation has also received considerable attention. In [72] Simo and Vu-Quoc applied the model to study the dynamics of earth-orbiting flexible satellites with multibody components. It has also been used for studying the dynamics of flexible mechanisms [7,10], robotic technology [78], the coupled geometric and constitutive nonlinear response of structures and buildings in the static [56] and dynamic cases [57], including applications to passive control in earthquake engineering [73,53]. The application of the model to the study of slender structures made of composite materials has been carried out in [75–77].

The formulation of time integrators for the Reissner–Simo theory of elastic rods results to be a particularly challenging task since the model describes a dynamical system evolving on a nonlinear manifold rather than on a linear space. Several approaches have been proposed in order to design time integrators respecting the geometry of the configuration space. The design of structure-preserving algorithms for dynamics of rigid bodies is closely linked to the design of new methods for elastic rods. In this sense, Simo and Wong [71] developed a midpoint algorithm on $SO(3)$[2] which conserves the total

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[1] An alternative to geometrically exact models is given by the *co-rotational approach* for rods, see for example [15].

[2] The symbol $SO(3)$ denotes the non-commutative group of proper rotations [47].

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