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VARIATIONAL FORMULATION OF DISCONTINUOUS-GALERKIN TIME INTEGRATORS

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Abstract. *Variational integrators provide a way to design structure-preserving time integrators for problems presenting a Lagrangian structure. The basic idea consists in obtaining algorithms from a discrete analogue of Hamilton's variational principle. Then, the discrete trajectories are stationary points of a discrete analogue of the action functional. The resulting methods enjoy a number of remarkable properties: i) they exactly conserves the momenta associated to the symmetries of a discrete version of the Lagrangian, ii) they define a discrete symplectic flow on the phase space and iii) they show an error in the total energy that remains bounded for exponentially long periods of time. A particularly interesting family of such methods is given by the so called Galerkin variational integrators. Their construction is based on approximating the trajectory of the system by means of piecewise continuous polynomials and providing suitable quadrature rules to approximate the action functional. Then, increasing the order of the interpolating polynomials and the accuracy of the quadrature rules allow to obtain higher order time integrators. In this work we extend the Galerkin methods to the discontinuous case yielding to a family of discontinuous-Galerkin (dG)-methods. To this end, we resort to using two key ingredients: 1) the trajectory of the system is approximated by means of piecewise polynomials which may presents a finite number of discontinuities across time interval boundaries and 2) we approximate the velocity of the system by means of an appropriate dG-time-derivative of the trajectory following some ideas presented in [1, 2] for static problems in elasticity. The resulting algorithms corresponds to a family of discontinuous-symplectic Runge-Kutta methods.*